

2. A. Yu. Ishlinskii, "Longitudinal oscillations of a rod when the after effect and relaxation follow a linear law," *Zh. Prikl. Mekh.*, **4**, 1 (1940).
3. V. S. Postnikov, "The problems of the damping of the oscillations of a cylindrical specimen," *Fiz. Met. Metalloved.*, **6**, No. 3 (1958).
4. A. P. Aleksandrov and Yu. S. Lazurkin, "Study of polymers. I. Highly elastic deformation of polymers," *Zh. Tekh. Fiz.*, **9**, No. 14 (1939).
5. P. M. Gorbunov, "Solution of a problem of the motion of viscoelastic materials," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 6 (1974).
6. O. Yu. Sabsai, M. A. Koltunov, and G. V. Vinogradov, "Analytical description of the creep of polymers in the flow state in the linear deformation region," *Mekh. Polim.*, No. 4 (1972).
7. G. M. Bartenev, N. P. Zoteev, and N. V. Ermilova, "Features of the highly elastic behavior of rubber-type polymers for small rates of deformation," *Mekh. Polim.*, No. 4 (1974).
8. A. N. Tikhonov and A. A. Samarskii, *The Equations of Mathematical Physics* [in Russian], Nauka, Moscow (1966).

STRENGTH OF SINGLE-LAYER AND MULTILAYER
CYLINDRICAL VESSELS LOADED INTERNALLY
BY PULSES OF VARIOUS LENGTHS

V. A. Batalov, A. G. Ivanov,
G. G. Ivanova, V. N. Mineev,
V. N. Sofronov, and V. I. Tsypkin

UDC 620.178.7

Technological processes and experimental research involving the detonation of explosives must ensure the containment of the detonation products, the safety of personnel, and the protection of equipment. Frequently the test conditions impose rigid requirements on the size and weight of the vessels designed to localize the effect of pulsed loads of various kinds. Pressure vessels for operation under static loads, where the controlling parameter is the pressure, are commonly of multilayer construction. As a consequence of the stopping of cracks in the separate layers [1, 2] this construction can increase the level of the working pressure and avert the catastrophic rupture of the whole structure. It is of interest to investigate the behavior of multilayer shells under pulsed loads of various duration.

We have investigated the effect of an internal explosion on closed cylindrical vessels with an outside radius R_0 and a total wall thickness δ_0 (Fig. 1) filled with air at normal atmospheric pressure.

The difference in duration of the pulsed loading was achieved by different schemes for loading the vessel walls. In the first case (Fig. 1a) the vessel walls were loaded by detonation products, and in the second (Fig. 1b) by the impact of a thin auxiliary shell 3 accelerated by the detonation products.

The vessels were made of Kh18N10T austenitic stainless steel (as received) and had the same geometry but differed in the design of the casing 1. A single-layer casing was made of tubular stock (GOST 5632-61) and had one welded joint along a generator of the cylinder. The casing was joined to a seamless core 2 of thickness $\delta^1 = 2$ mm (Fig. 1) with no gap between them. A multilayer casing was made by winding five layers of millimeter sheet (1 GOST 3680-71) of width $4R_0$ onto the core as in [1] with negligible clearance between layers; the inside and outside ends of the sheet were welded. The covers of the vessels 4 were attached with twelve M 16 bolts. The initial mechanical properties of the materials of the single-layer and multilayer casings determined on samples of identical dimensions under steady tension at a strain rate of $5 \cdot 10^{-4}$ /sec are listed in Table 1.

A spherical explosive charge of radius r (50 wt. % TNT, 50 wt. % hexogen, density 1.65 g/cm³) was placed at the center of the vessel. The charge was fired from the center. All the experiments were performed at

TABLE 1

Material	Yield point $\sigma_{s(0,2)}$, kgf/mm ²	Tensile strength σ_b , kgf/mm ²	Rel. elongation δ , %
Kh18N10T Single-layer casing	41	65	40
Kh18N10T Multilayer casing	34	69	40

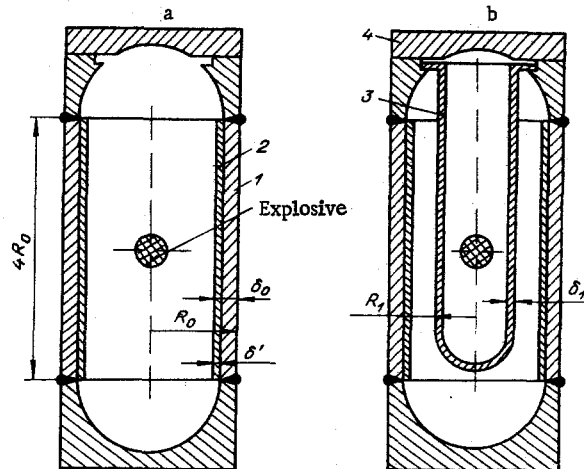


Fig. 1

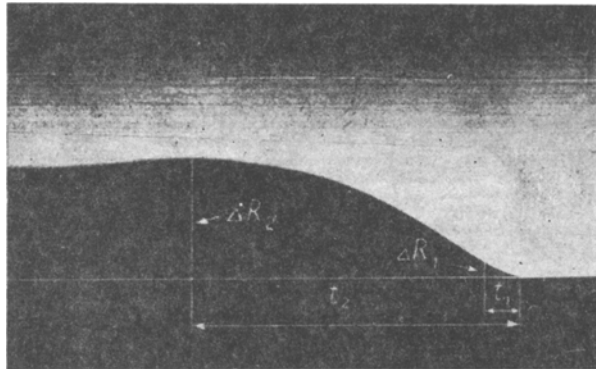


Fig. 2

temperatures between 2 and 18°C in vessels which had not been loaded previously. The nearest values of the mass m of the charges for which the vessels were ruptured and not ruptured were determined. A vessel was considered ruptured if at least one through crack was observed in its walls after the experiment.

A photographic recording method [3] was used to investigate the time dependence of the radial deformation of the central cross section of a vessel, the plane perpendicular to the axis of the vessel passing through the center of the explosive charge. A typical photograph taken by the moving-image camera is shown in Fig. 2 ($t_2 = 230 \mu\text{sec}$, $\Delta R_2 = 16 \text{ mm}$, experiment 4, Table 2). In addition, the final deformation of the vessels was recorded by coordinate grids drawn on their outer surfaces. The following quantities were determined in the experiments: The time t_1 from the beginning of the displacement of the vessel wall to the instant when it reached its maximum velocity v_0 ; the radial strain ϵ_1 at t_1 , where $\epsilon = \Delta R/R_0$, $\Delta R = R - R_0$, and R is the running value of the outside radius of the vessel; the time t_1 from the beginning of the displacement of the vessel wall to the instant it stopped moving or was ruptured; the radial strain ϵ_2 at t_2 . When the vessel was ruptured the values of ϵ_2 were determined more accurately by using the coordinate grids. The final radial and circumferential strains in the central cross section of unruptured vessels determined from the photographic recording mea-

TABLE 2

Exp. No.	Loading scheme	R_0 , mm	δ_0/R_0 , %	Form of casing	R_1 , mm	δ_1 , mm	m , g	r/R_0	v_0 , m/sec	t_1 , μ sec	ϵ_1 , %	t_2 , μ sec	ϵ_2 , %	T , μ sec	ϕ	State of vessel after experiment			
1	A	99,5	7,03	Multilayer	57	2	203	0,31	80	20	1,5	150	8	200	0,03	Not ruptured			
2							297	0,35	125	25	1,7	—	15	0,045	Ruptured				
3		100	7,5	Single-layer			203	0,31	80	20	1,5	160	9	160	9	180	0,03	Not ruptured	
4							207	0,35	125	25	1,7	—	16	0,045	Ruptured				
5	B	100	7,5	Single-layer	57	2	245	—	—	—	—	—	7	—	—	Ruptured			
6							160	—	—	—	—	—	6	—	—	—	—	—	—
7		99,5	7,03	Multilayer			57	2	203	450	—	—	—	—	—	—	—	—	Ruptured
8							29	0,3	160	—	—	—	8	—	—	—	—	—	—
9	—	—	—	29	0,3	120	—	—	—	—	—	—	—	—	—	Onset of rupture			
10				29	0,3	110	—	—	—	7	—	—	—	7	—	—	—	Not ruptured	
11	Test of auxiliary shell only. $\delta_1/R_1 = 3,5\%$																		
12	—	—	—	—	57	2	203	$r/R_1 = 0,54$	800	20	15	50	50	—	0,24	Ruptured			

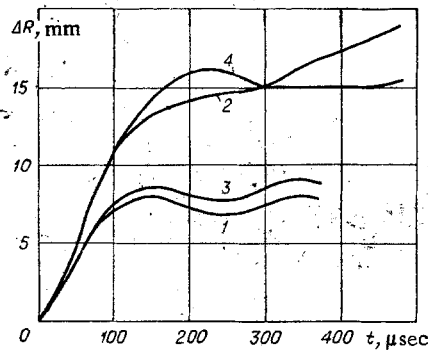


Fig. 3

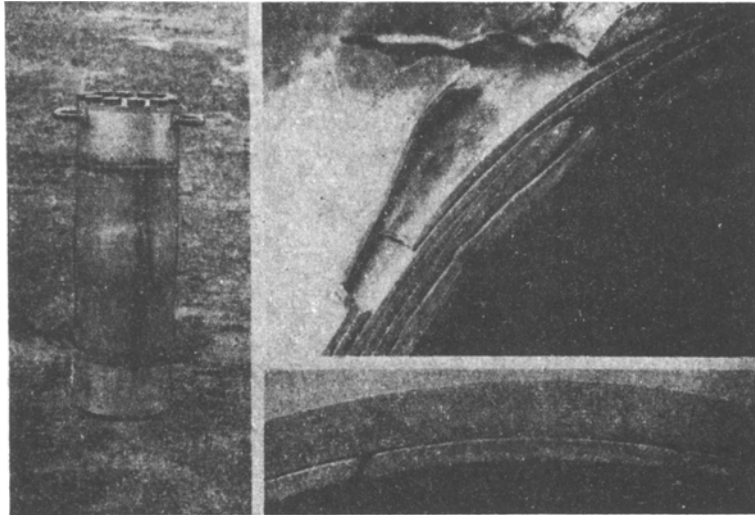


Fig. 4

measurements and the coordinate grids agreed. The values of v_0 were determined by differentiating the experimental relations between ΔR and t . In some experiments the periods T of radial vibrations of the vessels were determined also.

The error in the measurements of the time intervals was estimated as $\pm 3 \mu\text{sec}$, and that of the absolute displacements ΔR as $\pm 0.5 \text{ mm}$. The errors in the determination of the other quantities were the following: T , $\pm 10\%$; v_0 , $\pm 10\%$; t_1 and ε_1 , $\pm 15\%$; t_2 and ε_2 , $\pm 5\%$ when the vessels were not ruptured, and $\pm 10\%$ when the vessels were ruptured.

The experimental results are shown in Table 2. The quantity φ measures the energy of the explosive charge which is absorbed by the vessel walls in the region of the central cross section. As in [3] φ is defined as the ratio of the kinetic energy of a spherical layer of the material of the vessel of radius R_0 and thickness δ_0 moving with velocity v_0 , to the energy of the explosive charge.

Figure 3 shows the experimental relations between ΔR and t obtained in identical experiments by scheme A; the numbers on the curves correspond to the numbers of the experiments in Table 2. Figure 4 shows the external form of a vessel with a multilayer casing after experiment 9, and parts of the transverse cross section in the central section of this vessel, illustrating the nature of the failure ($m = 120 \text{ g}$). Figure 5 shows a fourfold enlargement of cleavage failure in a single-layer casing after experiment 6 with scheme B.

According to [4, 5] the duration of the pressure pulse in the forward wave for the values of r/R_0 used in loading the vessels by scheme A is $\tau_A \sim 100 \mu\text{sec}$.

When the vessels were loaded by scheme B the duration of the pressure pulse $\tau_B \sim 0.5 \mu\text{sec}$ was taken as twice the time for the shock wave to traverse the thickness of the auxiliary shell which collides with the vessel wall.

We note that $\tau_A \gg \delta_0/c$ and $\tau_B < \delta_0/c$, where c is the speed of sound in the vessel casing.



Fig. 5

When the vessels were loaded by scheme A the walls were accelerated for a time $t_1 \sim 20 \mu\text{sec}$, which is commensurate with the time t_2 . After reaching the velocity v_0 the vessel walls moved radially at a practically constant velocity up to $t \sim 80 \mu\text{sec}$, quickly came to rest, and then vibrated (Fig. 3).

When the vessels were loaded by scheme B the walls acquired an initial velocity v_0 as the result of the impact of the shell which was accelerated by the detonation products. The shell with $R_1 = 57 \text{ mm}$ and $\delta_1 = 2 \text{ mm}$ was ruptured directly before it collided with the walls of the vessel (experiments 11 and 12, Table 2).

When the shell with $R_1 = 29 \text{ mm}$ and $\delta_1 = 0.3 \text{ mm}$ was used, erosion craters 1-3 mm deep and 1-4 mm across appeared on the inner surface of the vessel walls. This may indicate the impact of a cloud of fine fragments.

When the vessels were loaded by scheme A the deformations of the one-layer and multilayer casings were of the same nature (Fig. 3) but the casings failed in different ways. In one of the experiments cracks stopped at the boundaries of the multilayer casing (Fig. 4), but a one-layer casing always failed with the formation of the through cracks.

In the experiments cited plastic deformation was observed in the central part of the vessels extending about $2R_0$ in the axial direction. The plastic deformation of the vessel walls was entirely in the circumferential direction. Measurements of the coordinate grids showed no plastic deformation of the vessel walls in the meridional direction.

The charge to rupture a vessel in scheme A was almost twice as large as that required in scheme B (experiments 2, 4, 6, 9). The deformations of the vessels before rupture when loaded by scheme A were more than twice as large as when loaded by scheme B (experiments 2, 4, 5-9). The decrease in the mass of the explosive charge to cause rupture can be related to two factors:

a) the increase in φ with a decrease in the gap between the shell and the explosive charge. For example, in changing from scheme A to scheme B for $m = 203 \text{ g}$, φ changes from 0.03 (experiment 1) to 0.24 (experiment 12);

b) the change in material properties under large loads. Thus, according to [6], the relative elongation of Kh18N10T steel after explosive loading by a pressure of $150 \cdot 10^8 \text{ N/m}^2$ decreased to 10-15%. A similar effect probably occurs in our case also, since in changing from scheme A to B the pressure in the vessel walls increases substantially. An auxiliary shell with $R_1 = 57 \text{ mm}$ and $\delta_1 = 2 \text{ mm}$ in experiments with scheme B produces a substantial increase in the effect of detonation products in the axial direction and leads to the separation of the covers for smaller explosive charges (experiments 3, 4, and 6, 7). The shell with $\delta_1 = 0.3 \text{ mm}$ did not show this effect (experiments 8-10). Under certain conditions the impact of the auxiliary shell on the vessel walls produces cleavage failure in them (experiments 5, 6, Fig. 5).

One-layer and multilayer casing show the same dependence of ΔR on t for $\tau_A \gg \delta_0/c$ (curves 1 and 3, Fig. 3) and have approximately the same strength (experiments 1-4). The slight difference in the time dependence of ΔR observed in experiments with one-layer and multilayer casings (curves 1 and 3, Fig. 3) is accounted for by certain differences in their material properties.

For $\tau_B < \delta_0/c$ the nature of the deformation of one-layer and multilayer casings is essentially different. In a multilayer casing the impulse is imparted mainly to the outer layers which may be ruptured while the remaining layers experience smaller deformations. A one-layer casing receives the whole transmitted impulse, and in the absence of cleavage the whole casing is deformed.

Thus, the behavior of vessels under internal dynamic loading is very dependent on the pulse duration, and the substantial differences between one-layer and multilayer casing must be taken into account in choosing types of structures and methods for testing their strength.

LITERATURE CITED

1. Technology and Equipment for Designing Pressure Vessels [in Russian], NIIFORMTYaZhMASH, Ser. 2, No. 14 (1972).
2. N. J. Petch, "Thin laminates can arrest fracture in pressure vessels," *Engineer*, 229, 51 (1969).
3. A. G. Ivanov, L. I. Kochkin, L. V. Vasil'ev, and V. S. Kustov, "Explosive rupture of tubes," *Fiz. Goreniya Vzryva*, No. 1 (1974).
4. M. A. Sadovskii, "The mechanical effect of blast waves in air," in: *Physics of Explosions* [in Russian], Izd. Akad. Nauk SSSR (1952).
5. V. V. Adushkin, "The formation of a shock wave and the dispersion of detonation products in air," *Prikl. Mekh. Tekh. Fiz.*, No. 5 (1963).
6. E. I. Bogdanovskaya, L. V. Dubnov et al., "The effect of the detonation parameters of explosives on the hardening of Kh18N10T steel," *Fiz. Goreniya Vzryva*, No. 5 (1975).

FORMATION OF STAGNANT ZONES IN VISCOPLASTIC MATERIALS ON THE CONVEX AND CONCAVE PARTS OF RIGID BOUNDARIES

E. M. Emel'yanov and A. D. Chernyshov

UDC 539.374

The flow of viscoplastic media [1] in channels with perturbed boundaries has been considered previously in [2, 3], using the small-parameter method, and the flow in channels of elliptical cross section has been studied and solved in [4].

1. The rheological relation for a Bingham viscoplastic medium has the form

$$\sigma_{ij} = (\sqrt{2k/V \varepsilon_{q1}\varepsilon_{q1}} + 2\eta)\varepsilon_{ij} + P\delta_{ij}, 3P = \sigma_{ii}, \quad (1.1)$$

where σ_{ij} are the components of the stress tensor, ε_{ij} are the components of the deformation rate tensor, k is the yield point, and η is the viscosity.

The deformation rate tensor is related to the components of the flow velocity vector of the medium v_i by Cauchy's equation

$$\varepsilon_{ij} = (1/2)(v_{i,j} + v_{j,i}).$$

In both problems, considered in this paper, only the component of the velocity vector v_z will differ from zero, and must be sought in a cylindrical system of coordinates in the form of a series in powers of the small parameter δ

$$v_z(r, \varphi) = v^0(r) + \delta v'(r, \varphi) + \dots$$

To formulate the problem in dimensionless form we will refer the stresses to the yield point k , the variable lengths to the radius of the tube R , and the velocity to the quantity kR/η . Then, Eq. (1.1) in terms of the dimensionless variables will take the form

$$\sigma_{ij} = (1 + I^{-1/2})2\varepsilon_{ij} + P\delta_{ij}, 3P = \sigma_{ii}, \quad (1.2)$$

where $I = 2\varepsilon_{ij}\varepsilon_{ij}$.

We will assume that the medium adheres to the surface at the rigid boundaries of the channels. In the flow regions the components of the deformation rate tensor

$$\varepsilon_{rz} = \frac{1}{2}(v_{,r}^0 + \delta v'_{,r}) + \dots, \varepsilon_{\varphi z} = \frac{1}{2}\delta \frac{1}{r} v'_{,\varphi} + \dots \quad (1.3)$$

will differ from zero.

Substituting (1.3) into (1.2), we obtain

Voronezh. Translated from *Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki*, No. 5, pp. 158-165, September-October, 1978. Original article submitted July 28, 1977.